

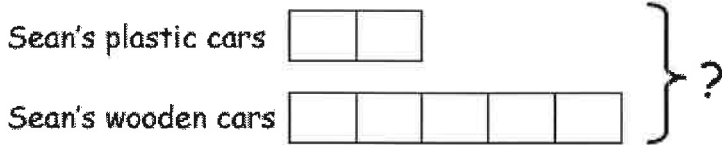
Addition Model Drawing Examples & Tips

Tips:

- You can make model drawing into a concrete activity by using manipulatives like pattern blocks or tiles to represent the units in a word problem.
- If your students are struggling with the numbers in the word problem, change them to simpler ones and start again. Then, go back to the more difficult numbers and see if the students are successful. If not, stay with smaller numbers.
- With any one word problem, there can be many models that you could draw to represent the words.
- For any one model you draw, there can be numerous ways to solve the problem.

Example #1

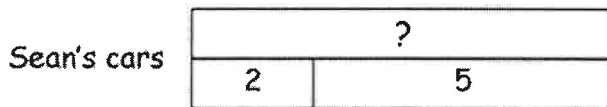
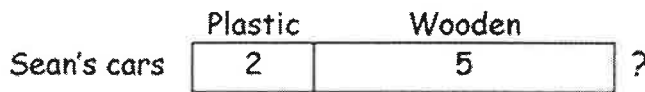
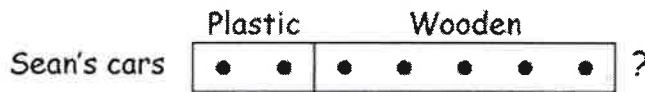
Sean has 2 plastic cars. He also has 5 wooden cars. How many cars does Sean have altogether?



$2 + 5 = 7$

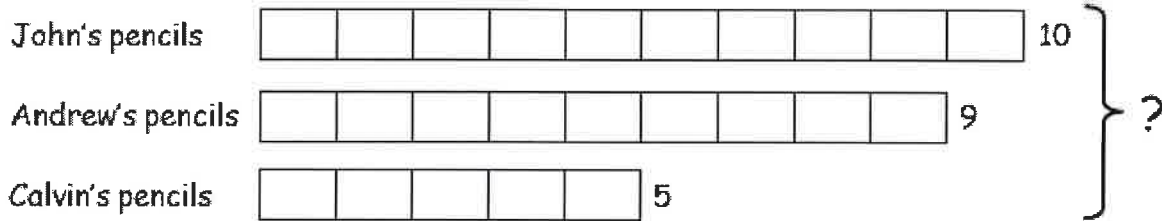
Sean has 7 cars altogether.

Other ways to solve this problem:



Example #2

John had 10 pencils, Andrew had 9 pencils, and Calvin had 5 pencils. They decided they'd put their pencils together and share them equally. How many pencils did each student get?

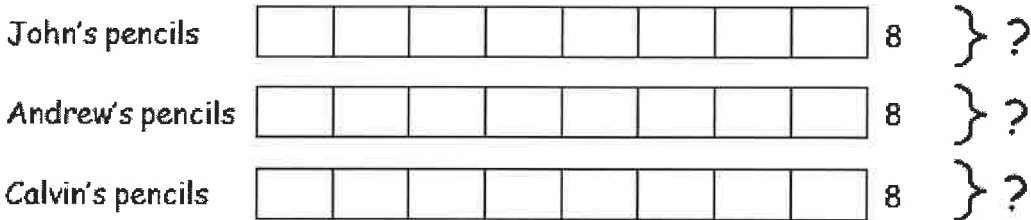


$$\begin{array}{r} 10 + 9 + 5 = \\ 19 + 5 = \\ \quad 1 \quad 4 \\ 20 + 4 = 24 \end{array}$$

$$\begin{array}{r} 10 + 10 + 4 = ? \\ 20 + 4 = 24 \end{array}$$

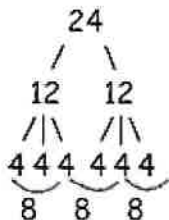
or

$$\begin{array}{r} 10 + 9 + 5 = \\ \quad 1 \quad 4 \\ 10 + 10 + 4 = 24 \end{array}$$



$$24 \div 3 = 8$$

or



John, Andrew, and Calvin received 8 pencils each.

Other ways to solve this problem:

	John	Andrew	Calvin	
Pencils	••••• •••••	••••• •••••	••• ••	?

John	Andrew	Calvin	
?	?	?	24

	John	Andrew	Calvin	
Pencils	10	9	5	?

John	Andrew	Calvin	
?	?	?	24

Example #3

Jesse had \$3.00 more than Clinton. If Clinton had \$10.00, how much money did they have altogether?

Jesse's money	\$10.00	\$3.00	} ?
Clinton's money	\$10.00		

$$\underbrace{\$10.00 + \$10.00} + \$3.00 =$$

$$\$20.00 + \$3.00 = \$23.00$$

Jesse and Clinton had \$23.00 altogether.

Subtraction Model Drawing Examples & Tips

Tips

- Most subtraction problems require you to draw a longer unit bar..
- It's really helpful to identify the segment of the unit you're subtracting and draw a diagonal slash (sometimes called a *vertical slash* even though it's not exactly vertical) through the value. This is a great visual reminder.

Example #1

Nathan had \$27.00 to buy gifts for his family. If he spent \$9.00 on a gift for his brother, how much money did he have left to spend on the rest of his family?

Nathan's money

?	\$9.00
---	--------

 \$27.00

$$\$27.00 - \$9.00 = \$18.00$$

or

Using Subtraction Compensation

Question: How can I make an easier problem?

$\$27.00 (+1.00)$	$\$28.00$
$-\$9.00 (+1.00)$	$-\$10.00$
	$\$18.00$

I can add \$1.00 to both the minuend and subtrahend to avoid regrouping.

Either way the answer is \$18.00

Here are two different types of models you can show students.

Part-whole Example 1:

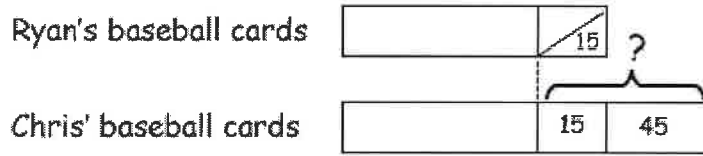
Nathan's money	$\$27.00$
spent	$\$9.00$
left	?

Comparison Example 2:

Nathan's money		}	\$27.00
spent	$\$9.00$		
left	?		

Example #2

Ryan and Chris started out with an equal amount of baseball cards. Ryan lost 15 cards, and Chris collected another 45 cards. How many more cards did Chris have in the end?



$$45 + 15 = 60$$

or

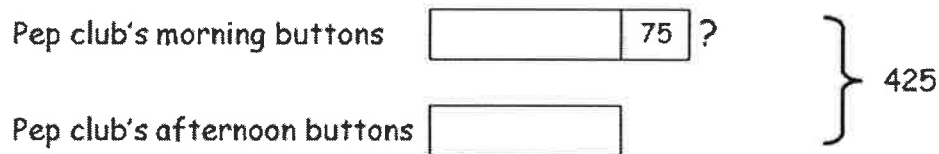
$$40 + 10 + 5 + 5$$

$$\begin{array}{r} \diagdown \quad / \\ 50 + 10 = 60 \end{array}$$

Chris had 60 more cards in the end.

Example #3

The pep club made 425 buttons to sell on Friday. The club sold 75 more buttons in the morning than they did in the afternoon. If all the buttons were sold, how many buttons did the pep club sell in the morning?



A. $425 - 75 = 350$

or

$$\begin{array}{r} 425 (+25) \quad 450 \\ - 75 (+25) \quad - 100 \\ \hline 350 \end{array}$$

B. $2 \text{ units} = 350$

$$\begin{array}{l} 1 \text{ unit} = 350 \div 2 \\ = 175 \end{array}$$

or

$$\begin{array}{l} (300 \div 2) + (50 \div 2) \\ 150 + 25 = 175 \end{array}$$

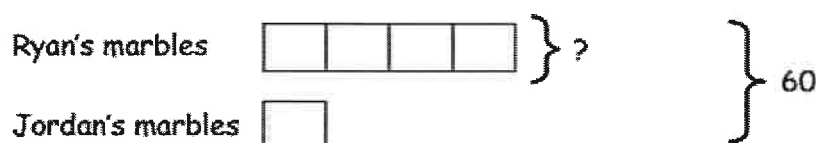
The pep club sold 250 buttons in the morning.

Multiplication Model Drawing Examples & Tips

Tips:

- When a problem says, "There were ____ times as many," hone in on what that means. Add one unit to your unit bar at a time, and count with students while adding each unit. (I call this the *counting method*. For example, if you have, "There were 4 times as many," say, "Okay, now we start with 1 times as many because our models are equal. Now we add 2 times as many (one unit bar), 3 times as many (another unit bar), and 4 times as many (a third unit bar)" so they can see how each unit makes an "as many." If we don't do it this way, students might see "4 times as many" and think they add 4 units to the base unit bar. That will skew the answer.
- For multiplication problems, it's usually helpful to draw a smaller unit bar to begin with. That way, you can add to it.
- The computation is where you can differentiate model drawing, allowing students to get to their answers in whatever way is easiest for them.
- Check to make sure that the model mirrors the sentence. This becomes particularly important as you get into more-complex problems.

Example #1 Ryan had 4 times as many marbles as Jordan. If they had 60 marbles altogether, how many marbles did Ryan have?



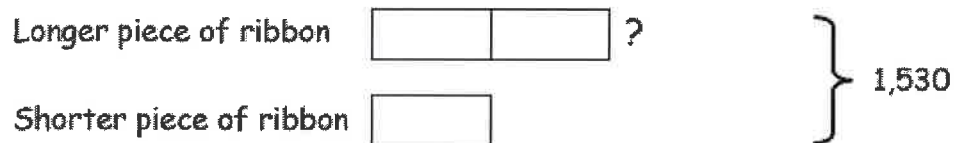
A. 5 units = 60
 1 unit = ?
 $60 \div 5 = 12$
 1 unit = 12
 4 units = 4×12
 48

B. 1 unit = 12
 4 units = ?
 $4 \times 12 = 48$
 or
 4×12
 \wedge
 $10 \ 2 \quad (4 \times 10) + (4 \times 2)$
 $40 \quad + \quad 8 = 48$

Ryan had 48 marbles.

Example #2

A ribbon that's 1,530 inches long is cut into two pieces. The length of one piece is 2 times the length of the other. What is the length of the longer piece?



A. 3 units = 1,530

1 unit = ?

$1,530 \div 3 = 510$

or

$(1,500 \div 3) = 500 + (30 \div 3) = 10$

$500 + 10 = 510$

B. 1 unit = 510

2 units = ?

$510 \times 2 = 1,020$ $(500 \times 2) + (10 \times 2)$

$1,000 + 20 = 1,020$

The longer piece of ribbon is 1,020 inches.

Example #3

Mike had 3 times as much money as Steve. Dick had 2 times as much money as Steve. If Dick had \$98.00, how much money did Mike have?

Mike's money ?

Steve's money

Dick's money } \$98.00

A. 2 units = \$98.00
1 unit = ?
 $\$98.00 \div 2 = \49.00

or

$\$98 \div 2 =$
 \swarrow \searrow
\$90 \$8
 $\$90 \div 2 = \45
 $\$8 \div 2 = \4
 $\$45 + 4 = \49
1 unit = \$49.00

B. 1 unit = \$49.00
3 units (Mike) = \$?
 $\$49.00 \times 3 = ?$
 $\$49.00 \times 3 = \147.00

or

$\$49 \times 3 =$
 \swarrow \searrow
\$40 \$9
 $(\$40 \times 3) + (\$9 \times 3)$
 $\$120 + \$27 = \$147$
3 units = \$147

Mike had \$147.00

Division Model Drawing Examples & Tips

Tips:

- With these problems, we show our units by dividing one long unit bar into its parts. So for example, if we have one unit that's equal to 10, and the problem asks us to divide it in half, we'll draw a vertical line through the center to show the halves.
- If the question asks for the value of one of the parts of a unit bar, go ahead and place your question mark right inside that section of the bar.

Example #1 28 chairs are arranged equally in 4 rows. How many chairs are there in each row?

Chairs

			?
--	--	--	---

 28

$$4 \text{ units} = 28$$

$$1 \text{ unit} = ?$$

$$28 \div 4 = ?$$

or

$$28 \div 4 = ?$$

/ \

$$20 \ 8$$

$$20 \div 4 = 5 \quad + \quad 8 \div 4 = 2 \quad 5 + 2 = 7$$

There are 7 chairs in each row.

Example #2

Eric had 3 times as many cookies as Sean. After Eric ate 50 cookies, he had half as many cookies as Sean. How many cookies did Eric have left?

Eric's cookies

--	--	--

Sean's cookies

--

After

Eric's cookies

--	--	--	--	--	--	--

50
┌───────────┐
│ │

Sean's cookies

--

Eric's cookies

--

 ?

Sean's cookies

--

$$5 \text{ units} = 50$$

$$1 \text{ unit} = ?$$

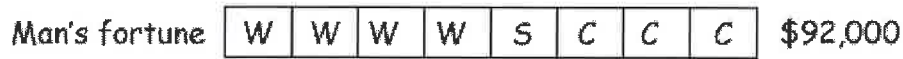
$$50 \div 5 = 10$$

$$1 \text{ unit} = 10$$

Eric had 10 cookies left.

Example #3

A man divided his fortune of \$92,000 into 8 equal parts. He gave 4 portions to his wife, 1 portion to his son, and divided the rest equally among three charities. How much more money did the wife receive than the son? (?)



- A. 8 units = \$92,000
1 unit = $\$92,000 \div 8$
= \$11,500
3 units = $3 \times \$11,500$
= \$34,500

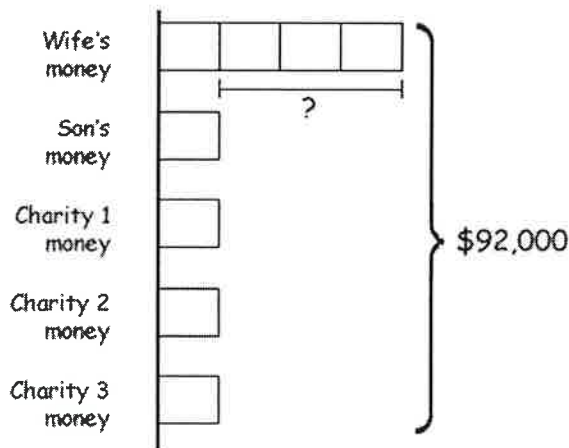
- B. 1 unit = \$11,500
4 units = ?
 $4 \times 11,500 = ?$
 $(4 \times 11,000) + (4 \times 500) = ?$
 $44,000 + 2,000 = 46,000$

- C. $46,000 - 11,500 = 34,500$
(wife's) (son's)

Another way to solve this problem:

A man divided his fortune of \$92,000 into 8 equal parts. He gave 4 portions to his wife, 1 portion to his son, and divided the rest equally among three charities. How much more money did the wife receive than the son?

Alternate Model:

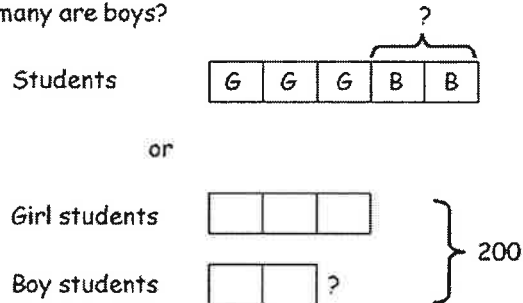


The wife received \$34,500 more than the son.

Fraction Model Drawing Examples & Tips

- Fraction problems offer us a unique choice in the way we set up unit bars. One way of doing it is by drawing a long unit bar (like we do in division) and sectioning it off to reflect fractional parts. The other way is to identify the variables (the parts of a whole) and draw separate unit bars for each.

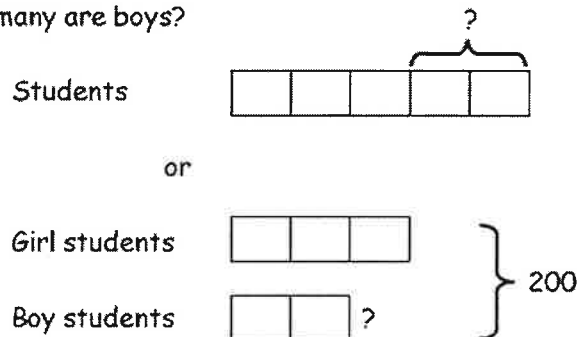
There are 200 students in the school. $\frac{3}{5}$ of them are girls. How many are boys?



See what I mean? These choices are great, and they give students the opportunity to do what makes the most sense to them, but they can also cause confusion. The message here is, "Offer a little guidance and a lot of practice!"

- When you see the words *remainder* or *remaining* in a fraction word problem, draw a long unit bar because most likely you'll be carrying down the remaining part into a new unit bar set. Here's an example problem with a remainder. At ABC Cookie Company, $\frac{3}{5}$ of the 300 bakers made sugar cookies. $\frac{2}{3}$ of the remaining bakers made gingersnap cookies. If the rest made chocolate chip, how many people made chocolate chip cookies? Here we'll need a long unit bar that we can divide into our fractional pieces.

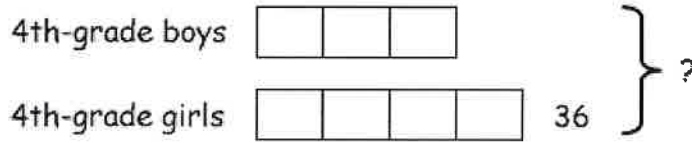
There are 200 students in the school. $\frac{3}{5}$ of them are girls. How many are boys?



- With fraction problems, we starting using the word *units* (abbreviated as *u*) in our computation. Then we focus on finding the base unit, or what 1u equals.

Example #1

In the 4th grade, $\frac{3}{7}$ of the students are boys. If there are 36 girls in 4th grade, how many students are there altogether?



A. $4 \text{ units} = 36$
 $1 \text{ unit} = 36 \div 4$
 $= 9$
 $7 \text{ units} = 7 \times 9$
 $= 63$

or

A. $4 \text{ units} = 36$
 $1 \text{ unit} = 36 \div 4$
 $= 9$
 $3 \text{ units} = 3 \times 9$
 $= 27$

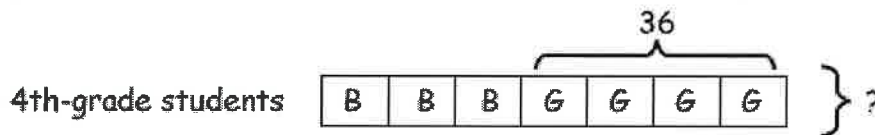
B. 27
 $+ 36$

 50
 13

 63

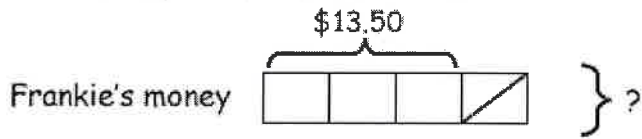
Another way to solve this problem:

In the 4th grade, $\frac{3}{7}$ of the students are boys. If there are 36 girls in 4th grade, how many students are there altogether?



Example #2

After spending 1/4 of his money, Frankie had \$13.50 left. How much money did he have at first?



A. 3 units = \$13.50

1 unit = ?

$\$13.50 \div 3 = ?$

Bright idea! Let's use partial quotient division

$$\begin{array}{r} 3 \overline{) 13.50} \\ - 12 \\ \hline 15 \\ - 15 \\ \hline 0 \end{array}$$

\$4.00

+ .50

4.50

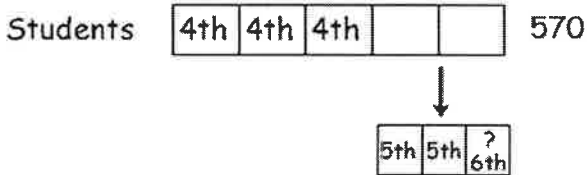
$\$13.50 \div 3 = \4.50

1 unit = \$4.50

B. 1 unit = \$4.50

$\$13.50 (3 \text{ units}) + \$4.50 (1 \text{ unit}) = \18.00

At Kenwood Middle School, 3/5 of the 570 students were 4th-graders. 2/3 of the remaining students were 5th-graders. If the rest were 6th-graders, how many 6th-graders were there?



A. 5 units = 570

1 unit = $570 \div 5$

= 114

2 units = 2×114

= 228

B. 3 sections = 228

1 section = $228 \div 3$

= 76